

## May 2017 subject reports

### Mathematics HL

#### Overall grade boundaries

##### Discrete

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 13	14 - 27	28 - 41	42 - 54	55 - 67	68 - 79	80 - 100

##### Calculus

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 12	13 - 25	26 - 38	39 - 51	52 - 64	65 - 77	78 - 100

##### Sets, relations and groups

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 13	14 - 26	27 - 40	41 - 54	55 - 67	68 - 79	80 - 100

##### Statistics and probability

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 13	14 - 26	27 - 40	41 - 53	54 - 66	67 - 79	80 - 100

To protect the integrity of the examinations, increasing use is being made of time zone variants of examination papers. By using variants of the same examination paper candidates in one part of the world will not always be taking the same examination paper as candidates in other parts of the world. A rigorous process is applied to ensure that the papers are comparable in terms of difficulty and syllabus coverage, and measures are taken to guarantee that the same grading standards are applied to candidates' scripts for the different versions of the examination papers. For the May 2017 session the IB has produced time zone variants of Mathematics HL Paper 1 and Paper 2.

## Higher level internal assessment

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 2	3 - 5	6 - 8	9 - 11	12 - 14	15 - 16	17 - 20

### The range and suitability of the work submitted

Some schools presented an interesting variety of explorations on topics of personal interest to students. It was clear that in these cases the students had received clear guidance from the teacher. Such schools should be commended.

On the other hand a significant number of candidates submitted what they themselves called a “research report” which simply consisted of transcribing mathematics and copying images from online sources or textbooks. An exploration that simply relays published findings is not likely to achieve high levels.

There was evidence to suggest that some schools are still advising students to write explorations on Mathematical topics that are well beyond the level of the Maths HL course. It is difficult in such cases for students to write an exploration that meets the aims of the IA within the page limit. As stated in the guide “The final report should be approximately 6 to 12 pages long. Students should be able to explain all stages of their work in such a way that demonstrates clear understanding. While there is no requirement that students present their work in class, it should be written in such a way that their peers would be able to follow it fairly easily.” Some explorations were still far too long.

### Candidate performance against each criterion

#### Criterion A

This year the highest achievement level in this criterion seemed to be inaccessible for a larger number of students. A number of students produced explorations which were far too long. Some students submitted work with little flow, segmented with sub-headings. For a piece of work of this length there is no need for a table of contents or a research question. In some cases, students produced a research report about mathematics beyond the scope of the course, and in doing so ended up writing a piece of work that read more like a chapter out of a text book rather than an exploration.

### Criterion B

Mathematical presentations were generally good. In most cases variables and parameters were defined and graphs were labelled. Unfortunately some candidates still presented work with calculator notation using \* for multiplication and ^ for powers.

### Criterion C

This criterion continues to present difficulties for some teachers and students. Some teachers award marks in this criterion, when a student simply introduced their exploration by some feigned interest, such as “I have been playing basketball since I was 4 years old”... Often these rationales were not supported by the rest of the work. Research reports of familiar “textbook” derivations cannot be awarded high levels unless the work is personalized and / or the student’s voice can be heard. Simply learning new mathematics does not demonstrate abundant personal engagement.

### Criterion D

Some students provided ongoing meaningful and critical reflection throughout the work. However, more students provided a summative reflection at the end of the exploration as part of the conclusion. Although this is not entirely wrong, the hazard in writing a reflection at the very end, is that students end up describing what was done, without providing any arguments about the validity or correctness of their approach. Reflection in explorations should be ongoing, and act as a stepping stone from one part of the exploration to another. Ongoing critical reflection is meant to drive the development of the exploration, by interpreting results, discussing the implications of results and possibly refining the approach taken when recognising shortcomings.

### Criterion E

The mathematical content in explorations varied greatly. There still seems to be confusion among teachers regarding what is “commensurate” with the course. The mathematics does not need to be exclusively from the section of the syllabus that is only HL. A student may use simple mathematics but apply it to a topic that is personalized and still obtain a good grade. If the mathematics used is very simple, then it cannot obtain high scores as it cannot be deemed to reflect the sophistication expected. On the other hand, students who choose to write research reports on topics that are well beyond the level of the course, often end up not being able to explain the mathematics from one step to another, making it difficult to gauge the level of understanding. Unfortunately a number of times, errors were found in students’ work that were not identified by the teacher.

## Recommendations for the teaching of future candidates

It is of fundamental importance that students cite any work at the point of reference in the exploration; this also includes any images, charts or diagrams.

It is recommended that the exploration is introduced early in the course, but the actual process should be delayed until a fair amount of the syllabus has been covered. Teachers should invest

time in going over the criterion descriptors with students to ensure that students thoroughly understand the expectations. One way of doing this would be to use explorations from the Teacher Support Material with students. There was evidence to suggest that students were not always given adequate feedback on a first draft. Students should also be advised to proof-read their work before submission.

Students should be reminded that the work submitted should be in standard format with an appropriate font and at least 1.5 line spacing. Using a small font and single spacing to fit an exploration into less pages should be avoided at all costs. Once the student work has been scanned it should be checked by both teacher and student to ensure that scans are in colour and that the scanned work is complete and legible.

Once the explorations have been submitted teachers need to mark the explorations. Evidence of marking must be shown on the submitted student work. This includes tick marks to indicate correct work, identification of errors, annotations and comments to explain where and how the achievement levels were awarded. The moderator's role is to confirm the teacher's marks but where annotations and comments are missing the moderator will have to mark the work without having any background information and very often it is less likely that a moderator can confirm all the achievement levels awarded. When annotating work digitally it is better for annotations to be made on the student work at the point of reference and not collected as an appendix or preface to the student work. Internal standardisation should take place to ensure consistent marking.

## Higher level paper one

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 13	14 - 26	27 - 37	38 - 51	52 - 64	65 - 78	79 - 100

### The areas of the programme and examination which appeared difficult for the candidates

- Poisson distribution
- Optimisation problems
- Complex numbers

### The areas of the programme and examination in which candidates appeared well prepared

- Functions
- Arithmetic and geometric sequences
- Differentiation and kinematics

- Curve sketching
- Integration by parts

## The strengths and weaknesses of the candidates in the treatment of individual questions

Question 1: Overall many wholly correct answers were seen to this question. However a significant minority made arithmetic errors in calculating the final term. A small number of candidates wrote out the entire binomial expansion without giving any indication as to which term was under consideration.

Question 2: In overall terms this was the question with which the largest number of candidates had the most success. Nearly all candidates were able to write down the range and find the inverse function. A small number were confused by what was required in terms of writing down the domain and range of the inverse function.

Question 3: Again many wholly correct answers were seen to this question. However many candidates used overly complicated methods to obtain the values of  $d$  and  $r$  which would have used up time. A number of candidates found  $r$  first and then found  $d$ . In this case they were still awarded full marks, but once again it often made the method of solution less efficient.

Question 4: This was a successful question for many candidates. For those who did not gain full marks the most common reasons were mistakes in the initial differentiation and giving answers in degrees rather than radians.

Question 5: This question was found to be a challenge for many students. Some clear, well thought through answers were seen, but a significant number of candidates embarked on speculative methods which either could not work or would only work after significant amounts of algebraic manipulation.

Question 6: The majority of candidates showed an awareness of what was expected in this question, but were let down by poor quality working. In a question where the final answer is given, they need to ensure they show all lines of working clearly.

Question 7: Better candidates were able to undertake the question with relative ease. However for weaker candidates there were a number of challenges. To be successful in part a) they had to correctly interpret the required mathematics from the question which was often not done well. In part b of the question the majority of candidates were unable to find the answer in terms of  $\ln 16$ .

Question 8: Overall the response to this question was pleasing. Most candidates were able to make a start on the question, although only a small minority were given full marks. Problems encountered in this question included using  $n = 1$  as the base case, not understanding the need of assuming the expression to be true, incorrect expansion and manipulation of the combinations and not fully understanding the implications of what they had done in terms of proving the final answer.

Question 9: Many fully correct answers were seen to this question and all students were able to make a reasonable attempt at the question. The majority of students successfully sketched the graphs required in part a), although the labelling was not always carefully done. In part b) most candidates recognised that integration by parts was the appropriate method, although a significant minority were unable to complete it accurately – the most common error was failing to take account of negative signs. In part c) most candidates recognised the use of the product rule, but as with part b) a significant minority were unable to complete it accurately.

Question 10: A number of wholly correct answers were seen to this question, but often errors were made. The fact that students had to interpret the situation caused some problems with some believing the perimeter of the window was the perimeter of the rectangle plus the perimeter of the semi-circle. In part a)(ii) a small number of candidates assumed the perimeter was a variable and also forgot to justify that the area was a maximum. In part b) a significant minority of candidates did not understand what was required.

Question 11: Part a) was well done by the majority of candidates although a number did not know how to reduce it to an equation in tan and others did not know the angle associated to a

ratio of  $-\frac{\sqrt{3}}{3}$ . Part b) was successfully completed by most candidates. A small number of

candidates seemed wholly unaware of compound angle formulae and suggested that  $\sin(A + B) = \sin A + \sin B$ . Part c) was a challenging question for nearly all students with only a very small number gaining full marks. However, it was pleasing to see that the question was accessible to all students even though the levels of success were varied. Many students were able to find the modulus but a number failed to manipulate the trigonometry to gain the correct answer. Finding the argument proved challenging with only a handful of correct answer in simplified form seen. Students often omitted the negative sign and for those that overcame that hurdle they were unable to simplify to gain the final answer. Many students gained marks for finding the cube roots due to the follow through rule being applied.

## Recommendations and guidance for the teaching of future candidates

- Students need to cover the entire syllabus.
- Students should be encouraged to pay attention to mathematical notation and accuracy.
- Teachers should emphasise the importance of students setting out their procedures in a logical fashion.
- Most of the questions in this paper used common problem solving strategies and this should be a focus for candidates.
- Students need to practice papers of a similar style in order that they understand the need to balance their time.
- Students need to be made aware of appropriate terminology.

On the whole candidates seem to have coped well with the paper with a small number gaining full marks for the paper and very few gaining marks in single figures.

## Higher level paper two

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 13	14 - 26	27 - 40	41 - 53	54 - 67	68 - 80	81 - 100

The areas of the programme and examination which appeared difficult for the candidates

- Vector manipulation
- Counting principles
- Curve sketching using GDCs

The areas of the programme and examination in which candidates appeared well prepared

- Probability, including tree diagrams
- Statistics, including use of  $E(X)$ ,  $Var(X)$
- Polynomial factorization
- Coordinate geometry

The strengths and weaknesses of the candidates in the treatment of individual questions

#### Section A

##### Question 1

This provided an easy start for the vast majority of candidates, with few incorrect answers seen.

##### Question 2

A relatively easy question, though one that posed problems for a number of candidates.  $m_1 m_2 = -1$  was sometimes not used, and a number of candidates differentiated the given

expression to obtain the incorrect  $8x + 2y \frac{dy}{dx} = 7$ .

A number of candidates used their GDC to obtain  $y = 0.433x + 1.30$ , which was acceptable, though some lost a mark through accuracy error(s) here.

Part b) posed a surprising number of problems. The volume formula is perhaps not as well known as it should be; the coefficient of the integral was often given as  $2\pi$ , or even conspicuously absent.

Some candidates used  $\pi \int x^2 dy$  and thus gained no marks. However, a good number of candidates did find the required value of 19.4.

#### Question 3

This was generally well done. A small number of candidates used (mistakenly) what they thought to be a continuity correction, and used 249 instead of 250 throughout their working. Evidence of clear working out should be encouraged, as a small number of candidates just wrote their answers straight down. If these were wrong, it was often the case that the full 7 marks were lost.

#### Question 4

Part a) in this question posed few problems. Those that were able to apply the cosine rule correctly to their triangle more often than not found  $c^2 - c - 12 < 0$  and were therefore able to use the result from part a) to write down  $-3 < AB < 4$ . Usually only the better candidates were able to deduce the final answer.

#### Question 5

This question was generally very well done and posed few problems except for the weakest candidates. Application of Bayes' theorem to part c) was sometimes seen, with varying degrees of success.

#### Question 6

This question proved to be a good discriminator. With careful presentation, many candidates were able to correctly manipulate the logs in the right hand side of the equation. Some stopped at that point, though the more able were able to work through to find the final answer of  $p = 2q$ . Some did make heavy work of this through using the quadratic formula, rather than factorising by sight.

#### Question 7

This question proved to be very problematic for most candidates, who seemed inexperienced with the general manipulation of vectors in this fashion, and correct answers were rarely seen. A number attempted to write  $a, b$  and  $c$  using coordinate vectors in  $\square^3$  and attempted to equate. Those that did so accurately, were often able to achieve the first two marks, but then made heavy work of being able to convincingly factor out a parameter. A number simply gave up after equating their vectors.



## Question 8

Not surprisingly, this proved to be a very difficult question. Correct answers were rarely seen, and of those that were, candidates generally applied the method  $16 \times 15 \times 14 \times 15!$ . Some candidates picked up a couple of marks through considering the number of arrangements of all papers, or indeed the non-science papers, though were unable to make much headway following this.

## Section B

## Question 9

This was thought to be a relatively straightforward start to section B, yet posed more difficulties than might have been expected. In part a) a number of candidates found the correct vector for  $\overline{BC}$  but did not proceed to find the equation of the line. It should also be emphasised again that it is required to see the correct notation for the line equation here, ie.  $r = \dots$ .  $BC = \dots$  was often seen, which lost the final mark.

Part b) was clearly and correctly answered by the better candidates. A small number used the same parameter when attempting to equate their equations and consequently gained little credit.

Parts c) to f) were often answered well, though some candidates often wrote  $\begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix} = \begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$  and

lost a reasoning mark here.

## Question 10

Part a) generally posed few problems for the majority of candidates, who were able to correctly write down a pair of simultaneous equations and go on to solve them, occasionally making use of the GDC.

Parts b) to f) were often answered very well, and candidates appeared well prepared for this type of question, especially when utilising the GDC.

## Question 11

Many candidates were able to make good progress with this question, except perhaps for the final part, thus showing it to be a good discriminator. Correct answers to part a) were often seen. Most candidates made good headway with part b), though with most usually utilising some form of long division, rather than the more straightforward method of equating coefficients.

Part c) posed the greatest issues. Perhaps many candidates had simply run out of steam by this stage, though correct curve sketches were rarely seen, with most missing the first couple of turning points at  $x = -1.23$  and  $x = -1$ . Candidates may also be helpfully reminded that coordinates here should be expressed to (at least) three significant figures. Of those that

sketched a correct graph, most went on to gain full marks in part d). However, such cases were not the norm, and many were left with a quadratic-type curve and thus only one follow-through mark was open to them in part d).

## Recommendations and guidance for the teaching of future candidates

- General presentation, particularly with regard to curve sketching.
- Practice with general manipulation of vectors.
- More difficult examples of logarithmic manipulation.

## Higher level paper three: discrete

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 7	8 - 15	16 - 25	26 - 30	31 - 34	35 - 39	40 - 50

### General comments

A good proportion of candidates seemed to find this to be an accessible paper with ample opportunities to demonstrate their reasoning skills and knowledge of the course algorithms.

### The areas of the programme and examination which appeared difficult for the candidates

Finding the general solution to a Diophantine equation.

Articulating clearly what is meant by a circuit and an Eulerian circuit.

Reasoning whether or not the complement of a graph  $G$  that has six vertices and an Eulerian circuit can also have an Eulerian circuit.

Verifying that a given second degree (order) recurrence relation is satisfied by  $u_n = A\alpha^n + B\beta^n$ .

### The areas of the programme and examination in which candidates appeared well prepared

Using the Euclidean algorithm to find the greatest common divisor of two numbers.

Finding a particular solution to a Diophantine equation.

Expressing two numbers as products of their prime factors and then determining the lowest common multiple of those two numbers.

Applying the nearest neighbour algorithm to find an upper bound for the travelling salesman problem.

Applying the deleted vertex algorithm in attempting to find a lower bound for the travelling salesman problem.

Using an auxiliary equation and its complex roots to solve a second degree (order) recurrence relation.

## The strengths and weaknesses of the candidates in the treatment of individual questions

### Question 1:

Question 1 was generally well answered. Most candidates were able to determine that 3 is the greatest common divisor of 264 and 1365 and that 120120 is the lowest common multiple of 264 and 1365. However, in attempting to determine the lowest common multiple, some candidates either did not complete the prime factorisation process or ignored the given instruction in the question and instead attempted to use  $\gcd(264,1365)\text{lcm}(264,1365) = 264 \times 1365$ . Some candidates thought that 91 was a prime factor of 1365.

In part (b) (i), most candidates accurately worked backwards to obtain a particular solution ( $x = 212, y = 41$ ) to the Diophantine equation  $264x - 1365y = 3$ . However, a number of candidates either did not attempt to find the general solution ( $x = 212 + 455N, y = 41 + 88N$ ) or gave a general solution that contained a sign error(s) due to not noticing the negative sign in the equation.

In attempting to find the general solution to the Diophantine equation in part (b) (ii) ( $264x - 1365y = 6$ ), candidates generally knew to multiply by 2 although some did it incorrectly and stated  $x = 424 + 910N, y = 82 + 176N$  as their general solution rather than  $x = 424 + 455N, y = 82 + 88N$  (or equivalent). In part (b) (ii), some candidates lost time by again working backwards rather than referring to the general solution obtained in part (b) (i). Follow through marks were often awarded in part (b) (ii).

### Question 2:

In part (a), most candidates were able to apply the nearest neighbour algorithm to find an upper bound for the travelling salesman problem for graph G. Errors included not returning to the starting vertex (A), inventing the 'furthest neighbour algorithm' or doubling the weight of a minimal spanning tree (which is not mentioned on the syllabus).

In part (b), a good proportion of candidates were able to apply the deleted vertex algorithm in attempting to find a lower bound for the travelling salesman problem for graph G. When applying

Kruskal's algorithm, marks were lost either by not indicating the edge order or by simply selecting an incorrect edge towards the end of the algorithm. Most candidates knew to reconnect vertex A with the two edges of least weight, namely, AB and AF. Some candidates lost time by repeating the method with other vertices deleted.

#### Question 3:

Responses to part (a) were quite varied with many candidates either not including all the required information in their definitions of a circuit and an Eulerian circuit, not expressing their definitions with sufficient clarity and precision or using the word 'path' instead of 'walk' or 'trail'.

Part (b), which required candidates to display sound reasoning and argumentation skills, was well answered by a reasonable number of the cohort. Candidates that started their proof by stating that the degree of all the vertices in  $G$  are even often progressed well. However, many responses were not precise enough with these candidates often confusing vertices and edges. A number of candidates gained partial credit for basing their argument on a specific graph  $G$  rather than developing a general argument. In such questions, it is important to realise that drawing one particular example of  $G$  does not constitute a complete and convincing argument.

In part (c), a pleasing number of candidates produced pairs of graphs with five vertices that have an Eulerian trail but not an Eulerian circuit. Errors included producing one correct graph and an incorrect complement or producing two incorrect graphs.

#### Question 4:

Part (a) was poorly answered by most candidates with only a small number even attempting to substitute for  $u_n$ ,  $u_{n+1}$  and  $u_{n+2}$  into the LHS of the recurrence relation. Many candidates seemingly did not understand what was required by the command term 'verify' and often just re-iterated the form that they knew the solution must have. Of the small number of candidates who correctly substituted into the LHS of the recurrence relation, most unfortunately developed solutions that were set to zero throughout and ended with the conclusion that  $0=0$ .

In contrast, part (b), which required the solving of a second degree (order) recurrence relation was quite well answered by a good number of candidates with many pleasingly obtaining full credit. Most candidates were not put off by the roots being complex. Errors committed were mostly of a computational nature rather than not knowing the solution method. Candidates who were awarded partial credit often found the correct auxiliary equation and its roots and then frequently obtained method marks for subsequent work.

## Recommendations and guidance for the teaching of future candidates

- The OR method in the markscheme is a neat way of tracking the linear combinations when applying the Euclidean algorithm.
- Encourage students to use technology to check the correctness of a particular solution to a Diophantine equation.
- Explain to students the importance of using the correct terminology and definitions when studying the graph theory part of the course.

- Provide students with questions, particularly in graph theory, that help hone their reasoning skills and provide practice in developing mathematical arguments.
- Provide students with opportunities to work on past IB papers and to discuss mark allocations with them.

## Higher level paper three: calculus

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 5	6 - 11	12 - 17	18 - 23	24 - 28	29 - 34	35 - 50

### General comments

This proved to be an accessible paper with the majority of candidates able to score good marks on questions 1 to 4. Question 5 proved to be more difficult and the reasons for this are discussed below.

### The areas of the programme and examination which appeared difficult for the candidates

The following topics caused most difficulties for the candidates: Riemann sums, manipulation of series and homogeneous differential equations.

### The areas of the programme and examination in which candidates appeared well prepared

The following topics were well done by the majority of candidates: Maclaurin series, L'Hôpital's rule, and the integral test

### The strengths and weaknesses of the candidates in the treatment of individual questions

Q1 Most candidates knew how to apply L'Hôpital's rule, and were able to use product and chain rule successfully to obtain the correct derivatives. In most cases it was correctly deduced that the rule needed to be applied a second time. Sometimes errors were made when trying to simplify the expression obtained which could have been avoided by substitution of 0 earlier.

Q2 a(i) was well done. A few students tried to obtain the expansion directly using Maclaurin's, this error could have been avoided if they had read the question carefully. In part

a(ii) some students were let down by their algebra, and chose to square each of the individual terms of the expansion for  $\tan x$  rather than the whole expansion.

a(ii) was an easy application of the results in a(i), and those who scored full marks in the earlier part invariably achieved all marks in the second.

Q3 Several students spent a lot of time trying to prove that  $f(x) = \frac{1}{x\sqrt{\ln x}}$  was a decreasing function. In general a result like this could simply be stated unless the question asks for it to be proved. In addition when the question gives the test that is to be used it may be assumed that the necessary conditions for the test are satisfied.

Candidates should be warned that they should not use the same variable in their integral as is being used in the series.

Most candidates were able to correctly integrate the expression. A few took the longer route of using integration by parts, but several of these were successful in their attempts.

In general, on the calculus paper, candidates should use limit notation when evaluating improper integrals.

To conclude the integral test candidates must not forget to comment that because the integral diverges then the series must do as well. Several simply wrote 'hence diverges' without making it clear if they were referring to the integral or the series.

Q4 (a) Most candidates were successful in differentiating  $y = vx$ . Some candidates failed to provide any evidence of integrating both sides in a "show that" question resulting in loss of final accuracy mark.

(b) This was done well. Many candidates scored some of the marks in this part. Evaluating  $\int \frac{1}{1+2v+v^2} dv$  was found to be the most difficult aspect.

Q5 (a) This part should have been very familiar to candidates via their work on Riemann Sums. Many though were unable to make the connection and so were unable to score any marks here.

(b) A correct solution to part (a) was not necessary for part (b) which relied on using the result given in the question. Many candidates did not take the hint of 'hence' and so did not sum the relevant parts of the result. Those that did write down the sum were usually successful in manipulating the log expression to obtain the correct result.

Part (ii) required careful thought. Changing the limits in the sum proved to be difficult for most of the candidates.

(c) Once again the candidates who took careful note of 'hence' were able to link the result in part (b) to the proof required in (c) (i)

Part (c) (ii) required linking back to part (a) which some of the students missed.

(d) Common errors here were to try and use results from tests for the convergence of series, or to say the sequence must converge to zero. It was not necessary for candidates to use formal language in their description of why it must converge.

## Recommendations and guidance for the teaching of future candidates

Teachers should advise candidates to read the questions carefully to improve the quality of response to the problem and write legibly as this will greatly help examiners with their marking.

Teachers must emphasize the need to show appropriate reasoning and clear methods/steps leading to the answer. Teachers should continue to emphasize the importance of command terms such as "show that", "Hence", "Explain" in the classroom and make sure candidate understand the meaning and expectation of these terms in the context of problem solving.

It was clear that some candidates were not familiar with the idea of Riemann sums, it is important to cover all parts of the syllabus.

Question 5 involved linking different parts and applying known knowledge in unfamiliar situations, it would be helpful to students to practice these types of questions under test conditions.

## Higher level paper three: sets, relations and groups

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 7	8 - 14	15 - 23	24 - 29	30 - 34	35 - 40	41 - 50

### General comments

Although this paper was very accessible, surprisingly, some candidates showed lack of familiarity with even the most basic ideas contained in this option. A significant proportion of candidates were very careless in the process of manipulation. This was especially evident in question 1. There were careless mistakes in parts of the question that based solely on prior knowledge. The layout of the proof of equivalence relation in Question 2 was sloppy. The algebraic manipulation in Question 3 was also weak. This is disappointing given this is such an accessible paper.

## The areas of the programme and examination which appeared difficult for the candidates

Candidates had some difficulty in applying the learned definitions to specific examples in 'show' and 'show that' questions. Although the definition of an equivalence relation and the properties of groups were well known, at times the definitions and properties were not interpreted correctly within the given examples.

Some candidates showed difficulties in using correct mathematical notation, particularly as pertains to equivalence relations and congruence. Some candidates also did not know how to determine the symmetric difference of two given sets.

Many candidates showed some difficulty in determining equivalence classes of a given equivalence relation.

Some candidates had difficulty in the algebraic manipulation necessary to show that a function in two variables was a bijection and determining its inverse.

Most candidates had difficulty in finding a proper subgroup of a given group, and in most cases were not even aware of the necessary conditions.

## The areas of the programme and examination in which candidates appeared well prepared

Candidates had good awareness of key definitions contained in this option. They generally showed good ability in answering questions on sets and set operations. They were familiar with properties of equivalence relations, definition of homomorphism and properties of groups, and could satisfactorily show that a given relation on a set was an equivalence relation, and a binary operation on a given set satisfied the group properties.

## The strengths and weaknesses of the candidates in the treatment of individual questions

### Question 1

(a)(i) Many candidates included the number 1 as a prime number in set B but surprisingly several candidates did not include 2 as a prime number. Some candidates struggled with the definition of congruence.

(ii) Some candidates obviously did not know the definition of the symmetric difference of two sets.

(b)(i) Students generally satisfactorily answered the questions in this part.

(ii) Some students did not verify the distributive property in the given case, as stated in the question, but rather attempted to justify the property in the general case, either using Venn diagrams or the double inclusion method.



## Question 2

(a)(i) Candidates were very familiar with the properties of an equivalence relation but in many cases they exhibited sloppiness at times in their notation for divisibility and congruence.

(ii) Some candidates showed difficulty understanding what an equivalence class is, and hence could not answer the question.

(b) Few candidates showed any working out for this question, although some did answer this satisfactorily. Many candidates stated that there were 7 equivalence classes.

## Question 3

(a) Many candidates could not apply the definitions of surjective and injective to functions in two variables. Among the candidates that could answer the question partially, the proof of injectivity was better attempted than surjectivity. At times candidates failed to state that both of these conditions are necessary to show that a function is a bijection. A number of candidates failed to score some of the R marks as their answers had no conclusion.

(b) Most candidates understood that they already found the inverse function in showing that the function is surjective, but sometimes they did not use correct notation. A number of candidates showed the misconception that they should interchange the variables  $x$  and  $y$  to obtain the inverse.

## Question 4

This question was attempted satisfactorily by most candidates.

(a) This part of the question was well attempted even for some weak candidates. The majority of them knew the definition of an Abelian group. Most candidates were able to show that the given binary operation on the given set satisfied the group properties. At times their notation was sloppy, and sometimes definitions were quoted without a correct interpretation in the given problem. A few candidates stated that the given binary operation was associative and commutative due to the properties of addition of real numbers. Some wrote down 0 as the identity and  $-a$  as the inverse since they inherited those from addition. A couple of candidates only proved commutativity of the binary operation instead of checking the general group properties.

(b) On this part most candidates knew the meaning of the order of an element and could make a start. Quite a lot of them jumped to the conclusion without completing their argument. The common reasons were no elements could be self-inverse and the only element with the given condition was the identity, hence contradiction. Some candidates failed to mention that the identity has order 1.

(c) Very few candidates answered this question successfully, and quite a few omitted it entirely. This was the worst attempted within the whole paper. Only a few managed to write down a proper subgroup of the given group. Among those who got the right answer, only a couple went on to justify their answer.

(d) Most candidates answered this question successfully. Even the weak candidates knew the definition of isomorphism although they struggled with the algebraic manipulation thereafter. A significant number of candidates wasted their time attempting to prove the function was bijection without noticing that was given in the question.

## Recommendations and guidance for the teaching of future candidates

Make students aware of basic facts like '1' is not a prime and '0' is not always the identity when addition is the operation.

Candidates should be exposed to different kinds of problems in which they need to interpret the properties of groups and equivalence relations, including how to find equivalence classes.

The use of correct communication and notation should be stressed, as well as all steps necessary in justifying their conclusions.

Candidates should be made aware of the need of being more rigorous in setting out proofs. Be harsh in scoring the details of equivalence relations proofs so that candidates learn the importance of precision.

Expose candidates to more examples of functions with more than one variable and how to prove injection and surjection in these cases.

Expose candidates to more examples of modular arithmetic.

## Higher level paper three: statistics and probability

### Component grade boundaries

<b>Grade:</b>	1	2	3	4	5	6	7
<b>Mark range:</b>	0 - 7	8 - 14	15 - 22	23 - 28	29 - 33	34 - 39	40 - 50

### The areas of the programme and examination which appeared difficult for the candidates

It was evident in Q2 that many candidates seem to confuse the probability density function  $f(x)$  and the cumulative distribution function  $F(x)$ .

Many candidates do not understand the Central Limit Theorem. A common fallacy is that as the sample size increases, the sampled distribution tends to normality. This is of course an impossibility

## The areas of the programme and examination in which candidates appeared well prepared

Most, but not all, candidates are able to use their graphical calculator to carry out hypothesis tests. Conclusions, however, are often not given in context when required.

## The strengths and weaknesses of the candidates in the treatment of individual questions

Q1 – Many candidates chose to calculate the mean and variance estimates using the formulae and then used the calculator software to carry out the  $t$ -test, apparently not realising that the software gave these estimates as part of the output. Division by  $n$  instead of  $n - 1$  was often seen so that 0.0066 instead of 0.0072 was often seen. Some candidates calculated the  $t$ -value using the formula and then found the  $p$ -value by using the cumulative probability function on the calculator. This is of course a valid method but it is more time consuming than intended. Many candidates failed to give the conclusion in context, as required by the question.

Q2 – It was disappointing to note that many candidates incorrectly integrated the cumulative distribution function to solve (a)(i) and (a)(ii). Some candidates attempted to calculate the median incorrectly by evaluating  $F(0.95)$ . Some candidates attempted to find the mean and variance of  $X$  by using integration by parts and some completed this successfully. This often required several pages of algebra. Candidates, however, were expected to evaluate the integrals using the integration facility on their calculator and most did that. Part (c) was poorly answered with many candidates stating that the distribution itself is approximately normal for large samples instead of the sample mean. It is extremely disappointing that what is arguably the most important theorem in Statistics is not understood by the vast majority of candidates.

Q3 – Parts (a) and (b) were well answered in general although in (a) some candidates failed to provide a convincing argument for summing an infinite geometric series. It is important in a 'show that' question not just to write down the answer without justification. The differentiation in (b) was disappointing in some cases with candidates using the method for differentiating quotients, think that  $p$  was a variable with derivative 1. Many candidates were unable to solve (c) successfully. A common error was to write

$$\text{PGF} = E(t^{2X+1}) = E((t^X)^2)E(t) \text{ instead of } E((t^2)^X)E(t)$$

which then enables the result from (a) to be used. Candidates who attempted to write down the series for the probability generating function of  $Y$  were generally more successful.

Q4 – Parts (a) and (b) were reasonably well answered although a certain amount of carelessness in algebraic manipulation was seen. In (c), some candidates used the calculator to minimise the variance but this method was unable to give the answers as fractions so that marks were lost.

Q5 – This was a slightly unusual question but it was well answered by many candidates. In (a), some candidates used  $r$  instead of  $p$  which was of course penalised. In (b), as in Q1, some candidates failed to give the conclusion in context as required. In (c), some candidates found

the inverse  $t$ -value of 0.177 instead of 0.823 which gave a negative value of  $t$  which resulted in a negative value of  $r$  which was however followed through.

## Recommendations and guidance for the teaching of future candidates

Many candidates seem to be unaware of the instruction on the front of the examination paper which states that 'Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures'. Many candidates lose marks by failing to obey this instruction.

Although candidates are generally competent in using their graphical calculators, not all candidates use them efficiently. Candidates should be aware that the output from carrying out a hypothesis test contains not only the value of the test statistic and the  $p$ -value but also the means and variances and degrees of freedom.

It would be useful if more time could be devoted to improving candidates' understanding of the Central Limit Theorem.